Chapter 2: Special Relativity

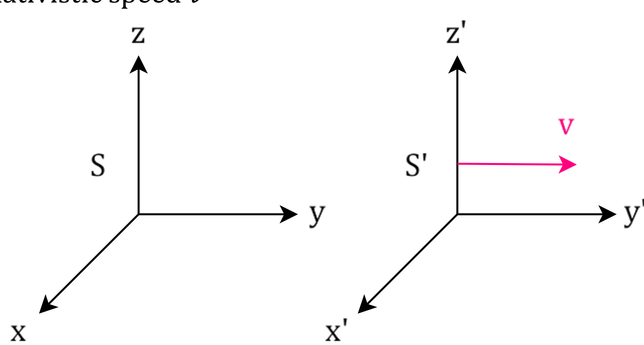
Special relativity is a theory that explains physical phenomena occurring when objects or reference frames move at speeds comparable to that of light ($c = 3 \times 10^8$ m/s). At such velocities, classical Newtonian mechanics becomes less and less accurate, leading to significant alterations in our predictions of space, time, and energy. However, because most of our experiences involve much lower speeds than that of light, relativistic effects are typically negligible, and classical mechanics suffices for practical purposes.

Despite encountering initial skepticism, technological progress over the years has provided substantial experimental evidence supporting the validity of special relativity, gradually diminishing opposition to the theory. One such example is the confirmation of **time dilation** through experiments with atomic clocks. Atomic clocks are incredibly precise timekeeping devices, and when synchronized and then compared after one has been in motion at high speeds (such as on airplanes or satellites), they show discrepancies predicted by special relativity. This effect has been observed in various experiments, including those conducted by researchers at the National Institute of Standards and Technology (NIST) and the European Space Agency (ESA).

2.3 Lorentz Transformations

The Lorentz transformation equations are important formulas that connect position and time measurements between two different frames of reference. These equations are based on Einstein's postulates and modify the classical relationships we already know. To relate observations from one frame to those in another, we refer to two representative frames: frame S and frame S'

S At rest
S’ Moving at relativistic speed v



The symbol v is reserved for the relative speed between the two frames, while u is reserved for the velocity of an object moving in one of the frames. To distinguish between quantities like position and time in each frame, we add ‘ marks to all variables in the S’ frame

Frame	Position	Time	Velocity
S	x	t	u
S’	x'	t'	u'

Classical Transformations

Classical, or "Galilean" relativity depends on time being the same in all reference frames, which gives the following equations relating position and time from one frame to another

$$x' = x - vt \qquad\qquad t' = t$$

We can take the derivative of the above equation to get the relationship for velocity

$$u' = u - v$$

At relativistic speeds (speeds approaching c), the classical transformations start to diverge from experimental observations. It turns out that (x, t) is related to (x', t') by a simple linear transformation. Skipping over the algebra, we find that the constant factors are all the same and we arrive at the following transformations

$$x' = \gamma_v(x - vt) \qquad\qquad t' = \gamma_v\left(-\frac{v}{c^2}x + t\right)$$

But what is γ_v? We call this constant factor the **Lorentz factor,** which increases as speed v approaches the speed of light

$$\gamma_v \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

Lorentz Transformation equations are the correct equations to use when relativistic speeds are involved. When $v \ll c$, known as the classical limit, these equations simplify to the classical transformations.

$$x' = \gamma_v(x - vt) \qquad\qquad t' = \gamma_v\left(-\frac{v}{c^2}x + t\right)$$

$$x = \gamma_v(x' + vt') \qquad\qquad t = \gamma_v\left(+\frac{v}{c^2}x' + t'\right)$$

Time Dilation is a consequence of special relativity. We use Δt_0 to denote the time elapsed in the frame in which two events occur simultaneously. The time Δt that passes between two events in another frame is longer than Δt_0 by a factor of γ_v

$$\Delta t = \gamma_v \Delta t_0$$

Length contraction is another consequence of special relativity. We use L_0 for the length of an object in the frame in which the object is at rest (usually called the **proper length**), and it could be in either frame S' or S, depending on the context. The length L as seen by an observer in another frame is shorter by a factor of γ_v

$$L = \frac{L_0}{\gamma_v}$$

2.5 Doppler Effect

The doppler effect happens with sound waves, but also with light waves

- Sound waves: **pitch** change.
- Light waves: **color** change

There is a formula to determine the change in frequency of the light **source** to an **observer** based on how fast the source (or observer) is moving

$$f_{\text{obs}} = f_{\text{source}} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c}\cos\theta}$$

Recall that light frequency is related to wavelength by the relation

$$f = \frac{c}{\lambda}$$

2.6 Velocity transformation

We saw how position and time change from the previous sections; Now we look at how velocity is affected. This also has implications for momentum, which are explored in the next section.

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \text{ and } u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

2.7 Momentum and Energy

Classically, momentum is the product of mass and velocity: $p = mu$. Relativistic momentum is similar but includes the Lorentz factor

$$\vec{p} = \gamma_u m \vec{u} \quad \text{where} \quad \gamma_u \equiv \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Total energy of an object moving at speed u.

$$E = \gamma_u mc^2$$

Internal energy of an object. Mass measures energy.

$$E_{\text{internal}} = mc^2$$

Kinetic energy.

$$\begin{aligned} \text{KE} &= \text{ energy moving } - \text{ energy at rest} \\ &= \gamma_u mc^2 - mc^2 = (\gamma_u - 1)mc^2 \end{aligned}$$

General formula relating energy, momentum, and mass of a particle. This is an important formula used in other parts of the text.

$$E^2 = p^2c^2 + m^2c^4$$